EXTENSIONALISM AND INDUCTION IN BOOLE[1]

NIMROD BAR-AM

Dibner Institute, MIT

38 Memorial Drive

Cambridge, MA 02139

U.S.A.

Web: http\\: www.nimrodbaram.com

ABSTRACT

Boole's discovery of the class Nothing is, perhaps, his greatest contribution to logic. It is a direct result of his attempt to narrow logic to the study of extensions. His unyielding extensionalism suggests a revolutionary rectification of the traditional Aristotelian conflation of logic, epistemology and ontology that had inhibited the growth of logic. Boole's extensionalism eliminates this traditional conflation by granting all terms equal legitimacy within logic, regardless of whether they depict, or even can possibly depict any objects. Logic and ontology were thus separated. The formal representation of a sharp distinction between necessary truths (tautologies) and informative, contingent, truths soon followed. Logic and epistemology were thus separated. This discovery has not been given its appropriate place in the learned literature. It is either completely ignored, or else it is noted as a minor result of Boole's partially successful attempt to express sentences as equations. Not so. The limiting of logic to the study of extensions underlies Boole's endeavor to justify induction.

CONTENTS:

1. PRELUDE: THE RISE OF THE CLASS NOTHING

> In accordance with a previous definition we may term Nothing a class. In fact, Nothing and Universe are the two limits of class extensions for they are the limits of the possible interpretations of general names, none of which can relate to fewer individuals than are comprised in Nothing, or to more than are comprised in the Universe. Now, whatever the class y may be, the individuals which are common to it and to the class "Nothing" are identical with those comprised in the class "Nothing" for they are none.
>
> George Boole, *The Laws of Thought*, 1854, p. 52.

A few months before Boole's introduction of a rudimentary version of the class Nothing, in the year 1847,[2] Augustus De Morgan introduced a rudimentary version of the theory of the complement class and of the universe of discourse (also known as "the universal class" or, in De Morgan's terminology, simply as "Universe"). De Morgan's aim was to generalize the theory of deductive inference to include inductive ones (which everybody identified with probability), and he took the urn from which one draws lots that serves as the basis for Bayes' theorem as his Universe. The idea of a universe of discourse is not new of course, indeed, with respect to probability theory it is rather old: it is the sum of all chances. However, De Morgan endorsed his notion of Universe as a basic logical concept and he insisted that, within this domain, every Universe is neatly composed of a pair of complementary terms (which he called "contrary names"). "Contrary names" he said, "with reference to any one universe, are those which cannot both apply at once, but one or other of which always applies".[3]

Now, the complement class of the Universe is the class Nothing, of course. (Later on, it was identified as the "empty class", or as the "empty set".) De Morgan did not fail to see this, but like many before him he deliberately refused to acknowledge that terms could have no reference. Thus, he prohibited terms to "fill" the universe so as to prevent their complement to become "empty". In order to achieve this he set the following maxim:

> "In future, I always understand some one universe as being that in which all names used are wholly contained: and also (which it is very important to bear in mind) that no one name mentioned in a proposition fills this universe, or applies to everything in it."[4]

Obviously, if a term may "fill" the universe, then, its complement is "empty". This is precisely what De Morgan was trying to rule out by his maxim. Consequently, he could not resist the traditional conflation of semantics, psychology and ontology in the style that Alexius Meinong rendered explicit later on: De Morgan followed tradition in demanding that all terms should have references, but since some terms obviously do not refer to real objects, he allowed some terms to refer to mere concepts (as Meinong did later too). This was his problematic version of the classical theory of reference that demanded conflation of ontology and logic, and inhibited developments. De Morgan's problematic theory of reference is nicely summed up in his *Syllabus of a Proposed System of Logic*.[5] De Morgan points there at what might be his greatest difficulty with terms that refer to nothing: he endorsed the traditional (Aristotelian) idea that the greater the extension of a term, the smaller its intension, and vice versa. And he says: "The name of greatest extension and of least intension, of which we speak, is the universe."[6] This would suggest that terms that refer to Nothing have the greatest intension. The problem was, I contend, precisely admitting the possibility that a class with no members has the greatest intension.

Clearly, the class Nothing is neither an object nor a concept. Boole explicitly allowed terms to "fill" his universe of discourse and, consequently, he allowed their complements to become "empty". His mere notation suggests a trivial proof that this "empty" class, the class Nothing as he called it, is nothing more than the complement class of the universal class. The proof is as basic as arithmetic can get. It is this: if the universal class = 1, then by definition of the complement class, it is 1-1, that is 0. End of proof. (Note the semi-conscious conflation of the class nothing with the number zero; the problem of admitting the class Nothing becomes no more, nor less, problematic than the problem of admitting zero as a number.)[7] Like zero, the class Nothing is a mere matter of (logical) convenience. The conflation of zero with the class Nothing, however, is not essential for the success of the proof. Indeed, it stands in the way of such a proof. Thus, the triviality of the proof is truly striking. It shows that what prevented De Morgan from admitting the class Nothing was no lack of talent for mathematical abstraction (since clearly he had such talent in abundance); it was a limited metaphysical horizon. Today we readily admit that in various senses the class Nothing has the greatest intension. For example we all admit that from contradictions anything follows, and we all admit that the empty set is a sub set of all sets.

In Boole's system one is led to refer to the class Nothing as the union of everything and nothing (i.e., as the class of individuals shared by the Universe and its complement class, Nothing). Thus, 1* (1-1) = 1*0 = 0. This is a special case of X*(1-X) = 0. The latter case is of course the more interesting and more general case: it states that the class Nothing is the conjunction of *any* class and its complement. Considering ways of proving the general case may have led Boole to formulate his famous, and striking index law, X*X =X, (without which the general case would not obtain, of course). The index law, as is well known, is fundamental to Boolean algebra.[8] The first logical system that sets ontological considerations apart from logical ones, thus came into being. The modern revolution in logic had begun.

2. Boole's place in the history of logic.

Boole performed one of the first important steps in the process of detaching logic from ontological considerations and from epistemology. He was the first to break the influence of Aristotelian essentialism upon logic. Traditional, Aristotelian logic was claimed to be a complete picture of the world that included, among other things, premises about reality and premises about secure knowledge. From these premises, meta-logical premises were deduced about the terms that could be used within logic, the propositions that could be used within logic, and the forms of inference with which logic is concerned. For example, traditional logicians often deemed informative assertions (such as "All men are mortal") as proven sentences, simply because they succeeded in deriving them from other informative assertions that they deemed "first principles" (for example "Everything that exists passes away"), or because their concepts (their concept Man, for example) seemed to imply it (by embedding the concept Mortal), or because, so they argued, the very act of observing a particular implies the universals to which it belongs[9]. This meant a conflation of informative assertions with nominal definitions;[10] and it became unclear whether the task of logic is the discovery of new informative truths or merely the semantic taxonomy of what was already known in advance. Hence, traditional logicians often found it difficult to deal with terms that did not designate (what was believed to be) essences, and they found it difficult, and sometimes even pointless, to study inferences from false premises: they could not distinguish properly between false sentences, contingent sentences, meaningless sentences and contradictions (indeed no on did so properly until Russell has resolved his paradox). Even inferences from mere conjectures were usually avoided; for fear that discussing them would be barren. This placed traditional logic in a position that was increasingly viewed as embarrassing: unlike modern logic, it was committed to the assumption that it is limited to propositions that express knowledge of the universe, knowledge of the natural kinds and types that the universe is made of, a detailed taxonomy of those natural kinds etc. And they also had to assume that this knowledge was irrefutable, and, somehow, a priori, as it was, part and parcel of the theory of logic.

The freedom to study all terms, those that seem to us natural and even those that do not make any sense to us, and the freedom to study necessarily or possibly false propositions is essential to modem logic. It is the freedom from premises about what is allegedly known a priori and about the conditions of knowledge. The discovery of this freedom was a gradual and laborious process. Here, I refer to it as the process of the detachment of logic from epistemology and from ontology[11]. Modem logicians carefully set apart methodology, epistemology, semantics and ontology. They render logic methodology proper. This is an enormous achievement that was concluded only in the works of Hilbert (on Geometry) and Tarski (on formal semantics). Here, Boole’s discovery of the class Nothing will be presented as a crucial stage in the progress towards that end.

It is interesting to note that from this aspect Boole’s theory is more modem and advanced even than Frege’s, since it rests on the recognition of the legitimacy of all possible premises, true, conjectural, or even contradictory. Even Russell, who was more advanced than Frege in this respect, is less clear than Boole on it, since like Frege he insists on presenting reductio ad impossibile as a conditional proposition ((if not-a then a) then a), and refuses to regard it as an inference, in the absence of a real theory of inference. (Unlike Tarski, both Frege and Russell accepted only one legitimate inference: the modus ponens.)

Today it is customary to look upon Boole’s contribution as an algebra. The original Boolean algebra (which was not a full fledged algebra at all) was amended by those who continued his work and who then incorporated it into the Frege-Russell logic as a small part of it. But Boole's merit lies, to repeat, not in the algebra he created but in the freedom he brought to logic, freedom from the epistemological and ontological limitations that Aristotelian logic has imposed on the development of logic. That is what this essay deals with at length.

My purpose here is to explain how modern logic took a significant theoretical step forward. I contend that it consisted of the termination of the influence of Aristotelian essentialism upon logic. This step was made possible by Boole's uncompromising extensionalism which allowed him to declare that any collection of objects is a legitimate class of objects, and in particular the class Nothing, which has no members. A second claim that I make here is that Boole's unyielding extensionalism was a means to an end: it was his strategy to reduce to bare bones the representation of the problem of induction, in an attempt to solve it. Of course he could not succeed in this overall mission. Therefore Boole stands in the history of logic, alongside Frege and Russell, as someone whose contribution to the development of logic was an inseparable part of his overall failure to surmount the insurmountable obstacles of epistemology.

3. Traditional logic is saturated with traditional epistemology and ontology.

As is well known, the aim of traditional epistemology was to describe knowledge — truths that are known with certainty, ways of acquiring such truths, and criteria for identifying them. These aims developed largely as responses to the claims of sophists and skeptics (real or imagined) that knowledge cannot be attained. Less well known, or less noted, is the historical fact that traditional logic was shaped by the aspiration to overcome these claims.[12] In order to achieve such an end traditional logic had to be anchored in ontology and be saturated with epistemology. It is easy to forget this fact today, when logic (with its various versions) is by design formal, and more reminiscent of computer language than an instrument that inspired such sublime pretensions. All the more so since the basic course in logic today is intended to teach a rather lackluster skill: the theory of valid inference. It is easy then to overlook, or to underestimate, the historical context that conflated the theory of valid inference with epistemology.

Let us briefly discuss dialectics as a case study: it is commonly considered to be the first art from which logic was developed. With Zeno it clearly serves as a tool for proving informative propositions about our world. Zeno, it will be remembered, used dialectics to prove Parmenides' theory by refuting its opposite. But with Socrates dialectics served only as a tool of refutation. This is an important difference of opinion: while Zeno identified refutation with proof of the negation of the refuted proposition, Socrates never viewed refutation as more than just that, refutation (and therefore he never claimed to know more than the knowledge of his own ignorance). Some sophists confronted both, and laid down the first important skeptical challenge to philosophy: they claimed (and demonstrated) that it was possible to prove and refute any proposition. If it is possible to first prove and then refute any proposition, then dialectics is completely worthless as a tool for searching for the truth and it certainly isn't a tool that guarantees absolute final positive results. How, then, if at all, is science possible?

Plato believed that there was no choice other than to give back to dialectics its Eleatic power as proof machinery. Concurrently, he launched a propaganda campaign: he dismissed the sophist critique and the sophists in general as troublemakers. In hindsight, it is easy to see that this was unjust and may have been more an expression of weakness than a suitable philosophical response. The sophists demonstrated that it is not easy to find the truth and may perhaps be impossible; to admit this means admitting that we must clearly and distinctly formulate what are the rules of searching for the truth (and of finding it). Plato never performed this task, at least not openly in his writings, and if he did so privately we know nothing about it. At most he demonstrated his own contribution to the development of dialectics— the diaeresis —in such dialogues as his *Sophist* and in his *Statesman***.** In hindsight, it is easy to see that Platonic diaeresis is not in any way a guarantee for absolute, final, positive results, and therefore not a suitable response to the critique of the sophist.

It seems that the first person who tried to openly confront the sophist challenge was Aristotle. One of his greatest merits here is his frankness and his admission of the process of his development. It is this frankness that allows him to openly admit that the sophists did produce a worthy challenge. It is his frank rivalry with Plato that enables him to expose Plato's failure to answer that challenge satisfactorily, and to attempt and produce his own answer, openly and clearly. Today we know that the skeptic's challenge cannot be answered. But Aristotle's attempt to answer it gave us the science we know call logic.

The father of logic acknowledges that the idea of it came to him as part of his attempt to provide a superior alternative to Platonic diaeresis, in order to train his students to deal with both the sophists on the one hand and the students of the Platonic academy on the other. As far as we know, his first books on the topic deal with the theory of refutation and a selection of rules of thumb for dealing with the sophists' quandaries. The central portion of his logic, however, is a formulation of several of the rules of deduction, described in his theory of the syllogism. This, clearly, is an unprecedented achievement. Its crowning glory is the use of linguistic variables as a substitute for terms. The use of variables turns Aristotle's logic into the first theoretical discussion of inference,[13] a discussion that focuses on logical form instead of concrete examples (in contradistinction to Zeno, Socrates, and Plato).

However, it is crucial to stress that despite Aristotle's ingenious and revolutionary use of variables as means for formulating his logic, the main concern of his logic remained epistemology. For example, the range of his variables is hardheartedly restricted by his essentialism. As a result, only a small number of all possible terms are admitted into his logic: those that are assumed to have essences as their reference.[14] And, strictly speaking, the main aim of his logic remains somewhat obscure. For example, sometimes it seems that his subject matter is not the valid inference, but rather the sound inference: inference with premises that are known to be first principles. Of course, dialectical refutations are discussed there too, but there is only a partial and confused symmetry between sound inferences and dialectical refutations in Aristotle; one that only partially resembles the modern strict symmetry between sound inferences and refutations.)[15] In fact, Aristotle often seems to consider logic to be useless unless first principles, conveying informative theoretical knowledge can be known with certainty[16]. Yet how can the truth of an informative theory be known? Most of the epistemological burden was placed on the propositions that are anachronistically termed the "premises" of a proper syllogism. They are not premises at all, but part time terms and part time definitions. They are referred to here as real definitions:

definitions that are both informative and necessary. A syllogism, according to Aristotle, is a pair of real definitions from which is derived a (slightly less necessary) definition (I will not discuss Aristotle's problematic contention that necessity can decline, which actually limits all reasoning to direct conclusions from a pair of real definitions). The peculiar standing of real definitions is what makes the leakage of epistemology into Aristotle's logic. Indeed, some of the passages generally deemed enigmatic, or even problematic, in the Organon**,** such as the opening and the ending remarks of the *Posterior analytics* are those devoted to explaining how real definitions are attained. Aristotle seems to explain the possibility of such knowledge in different ways almost every time he discusses it. Real definitions, he states, are found by means whose standing is not entirely clear since they seem to conflate strictly deductive processes (such as refutations) with processes whose standing is not entirely clear, but in retrospect they have been seen as extra-logical and as distinctively epistemological, such as the theory of induction, as well as his problematic theory of intuition, whose standing in his writings is partly logical and partly mystical.

The gist of Aristotelian logic, first and foremost, was to facilitate the presentation of a classificatory system (taxonomy) of (biological) reality. Definitions were the means to express this taxonomy. (Insisting that they are *real* definitions seemed like the means to insure that the taxonomy is not merely semantic, but rather that it represents reality accurately and exhaustively). Such complete taxonomy would ideally place at our disposal a sort of cosmic tablature, the blueprint of Creation. The tablature would be provided with compartments suitable for such or other essences and sub-compartments representing the properties that they necessarily encompass. This had a most curious result: any term seeking entry into a syllogism needed therefore to comply with strict preconditions, in order to ensure that we have a single taxonomy, a single classificatory system, when all is said and done. Terms, then, must depict essences, or else any proposition that includes them would be logically inadequate.[17]

How can we insure that our terms denote essences? According to Aristotle, the terms ‘man” and “wolf’ are legitimate since they denote essences; not so the terms “not-man,” “not-wolf,” “man-wolf”, and “man or wolf”. The distinction between them can be found in Aristotelian metaphysics, where we learn that there is no essence common to the class of not-wolves and the same holds for wolf-man. This is the source of the problem of admitting the class Nothing, and the many terms which denote it, into logic. Thus conceived, the theory of logic rested on ontological conjectures taken as justifiable in some manner (by a set of intuitions, or by a set of inductive generalizations, or by a set of dialectical refutations or by a set of common sense observations, or by a fusion of all, or some, of these poorly distinguished methods). The very point of Aristotle’s logic seems to depend on the truth of these ontological conjectures. This is why they are presented as definitions. To keep Nothing outside the realm of logic.

How is a real definition found? If its truth is unquestionable, how is it distinguished from merely nominal definitions? If it is an informative proposition, how is it distinguished from an incidental truth? A definition, in the modem sense of the word, is *any* equation (or, sometimes, an equivalence statement) expressing verbal identity. One side of the definition (the left one in English) is termed the definiendum and the other the definiens. Traditionally, however, a definition of this kind had been viewed as worthless just because it was merely verbal. Contrarily, tradition valued only the real definition, which allegedly explicates the nature of an entity by listing its essential (unique and irreplaceable) properties. The main difference between the traditional and modern theories of definition is with the role of definitions: the modern theory is permissive, unlike the traditional one. The role of the traditional theory is to determine necessary and sufficient conditions for a thing being an essence, while the modern theory allows any equation or equivalence statement to stand as a definition (some more, some less adequate, depending on the context, of course). This revolution is part and parcel of Boole's contribution to logic.

Consider the following classic examples for definitions: A. Man is rational animal. B. Man is featherless biped. On the assumption that both are true, how do they differ? Traditional theories of logic tell us that A is an example for a real definition and B is merely an example for an accidental definition. But what justifies this assertion? Aristotle states that men merely happen to have no feathers, but that of necessity they are rational. This is the traditional union of modal logic and essentialism: the rules for discovering necessary facts are declared to be the same as the rules for making real definitions. It is a circular justification, of course: we do not know how to discover necessary informative truths. It is, thus, important to emphasize that all definitions convey accidental truths and that out of the class of all accidental truths an elite class is (allegedly) chosen, (allegedly) delineated: the class of necessary informative truths, the class of real definitions. How, then, is it chosen? Clearly modal logic cannot do the work for us. Here traditional Aristotelian epistemology collapses, and traditional Aristotelian logic falls down with it.

Let me sum up this point: Aristotle, and more so the Aristotelian tradition, furnished logic with three very important, very influential and very problematic distinctions, 1. A distinction between legitimate and non-legitimate terms (or concepts). 2. A distinction between necessary and accidental truths. 3. A distinction between real and nominal definitions. Aristotelian essentialism aimed to secure these distinctions, for securing them seemed to make logic possible. (Incidentally, this is a germ of a transcendental proof.) Ultimately it meant that the skeptic's challenge could be answered, by a description of the method for discovering real definitions. From Aristotle to William Hamilton[18] logicians assumed that there was a logical solution to this problem. They expected that slow and careful logical clarification would explain why some (and only some) terms (or concepts) are legitimate, and which informative propositions are real definitions. In hindsight, it is easy to see why this was never achieved: logic had been designed to justify the status of real definitions, and an essentialist world-view was designed to serve as its foundation. Aristotle justified his logic by an appeal to a metaphysical framework that was presented as scientific knowledge (and as such, it was presented, as a consequence of his logic). He thus conflated logic and epistemology, semantics and ontology.

4. Detaching logic from traditional epistemology: Boole's extensionalism.

Boole' s outstanding contribution to logic is still under- appreciated because it took place against an outstanding metaphysical revolution that he championed and which is also still under-appreciated. Its core was the purging of logic of essentialism by means of unyielding extensionalism. Contrary to Aristotle, Boole declared that any collection of objects constitutes a legitimate class, and that all classes are equally legitimate subjects of logic. No longer is logic limited to terms that name essences, namely, natural kinds of tangible entities. Metaphysical, commonsensical and meta-biological criteria lose their import for the determination of the legitimacy of the terms that name classes. For the first time in history, then, the question of the legitimacy of classes or union of classes was rejected. His extensionalism, thus, enabled him to offhandedly break the monopoly of Aristotelian essentialism, to suggest such incredible ideas as the class Nothing, and to refine De Morgan's rudimentary version of the theory of the complement class and the universal class, in a manner that would give new meaning to De Morgan's famous laws of conversion (later known simply as De Morgan laws).

As every class is legitimate, a class defined by the use of an inconsistent criterion is legitimate too. For the first time in history, then, the class Nothing is admitted into logic, as are all the different terms that name it ("square circle", "the greatest prime number", "Goat-stag" etc.,). Thus, Aristotle's conflation of semantics and ontology has reached its timely end. If every term is legitimate, then terms formed by the addition of the word "not" (or the like) to any term are legitimate too, so the class of all objects not-in-a-class is legitimate too. It is its complement class. The complement to the class Nothing is the universal class, and he identified it with the universal class. There is an important philosophical message in these new, seemingly technical, notions.

This message, to my knowledge, was never presented properly. I now wish to present it.

For generations logicians divided truths into 3 categories: real, or essential, definitions (e.g., Man is a rational animal), accidental truths (e.g., Man is a featherless biped) and nominal definitions (e.g., "“Man” is “Rational animal”"). What is the differentiation between the first two? This question weighed heavily on logic. In hindsight it is easy to see that it is unanswerable: the propositions deemed real definitions are empirical conjectures just as those deemed accidental truths. Only nominal truths are logically necessary (later called “tautologies”). However, nominal truths were traditionally dismissed — as negligible and uninteresting, exactly because they were deemed empty of informative content. Thus, traditional logic was preoccupied with securing that which cannot be secured; its aim was to prove necessary informative sentences. Boole' s extensionalism and the clear-cut distinctions that it invited, allow us to realize all that.

In hindsight, and in a somewhat anachronistic mood, we may note that it was already Hume who stressed that necessary propositions cannot have content and that informative theoretical propositions cannot be logically necessary. He noted that induction, which everybody identified with the principal method for obtaining and securing informative theoretical knowledge, is a fallible method. Kant accepted Hume’s insight as his great challenge: he deemed the challenge as the demand to surpass Hume’s insight by devising a new logical means for proving informative truths. He ascribed to nominal definitions (which he called “analytical judgments”) classical logic, and to informative definitions (which he called “synthetic judgments”) he ascribed transcendental logic. It was an outstanding improvement: it rectified the Aristotelian latent conflation of nominal and informative truths, which is at the root of his first principles, his *real* definitions. And it did so in a manner that blocked the confusion or the conflation of the nominal (analytical) with the informative (synthetic).

The purpose of transcendental logic was to demonstrate the necessity of some informative judgments. Had Kant succeeded in this, we would have an interesting criterion for the distinction between necessary informative truths (proven by transcendental logic), necessary nominal truths (proven by standard logic) and accidental informative truths (informative judgments which cannot be proved). But, as is well known, the term “transcendental logic’, despite ingenuity and audacity, never denoted a logic: there were and are no transcendental axioms and rules of inference, not even their most rudimentary beginnings.

With Boole, for the first time logic displays a complete assimilation of Hume’s insight: the principle of extensionality unequivocally establishes that all and only tautologies are necessary truths. This insight is so elementary in hindsight that it takes the modern observer some time to understand what a giant leap it is. It may very well be that the triviality in hindsight of this great achievement is what prevented previous scholars from noticing its far from trivial results. Boolean formulas and inferences are open to diverse interpretations. Extensionalism led to the stress on this. Under Boolean analysis, propositions (or inferences) that superficially appeared different were revealed to share a common logical structure: they differed by their interpretations. For example, inferences whose validity is based on relations between nouns are represented by means of the same formulas that describe inferences whose validity rests on the relations between propositions. This is so because under different interpretations the same

Boolean formulas can serve to represent different facts. A basic yet prominent case of this kind is the proposition $x + (1-x)= 1$. It represents the universal class as the sum of any class and its complement. But it also represents the no less important proposition that a disjunction created by any proposition and its negation is a tautology.

This was a tremendous advance over traditional logic. Consider the following propositions:

“All citizens are loyal” and “All traitors are foreigners”. Both express the same content and are therefore expressed by the same Boolean formula. It is difficult to decide the issue within classical logic: intuitively, these propositions are obviously equivalent (on the understanding that we have here pairs of complementary terms). However, Aristotelian metaphysics divides all expressions into positive and negative. The expression “loyal” is positive, whereas the expression “traitor” is negative. It is therefore considered inferior and therefore unable to designate an essence. Thus, Aristotelian metaphysics ties the hands of traditional logic in order to ensure that only propositions of a certain type may be included in the results of Aristotelian epistemology. Indeed, it is very hard (and perhaps even impossible) to determine who really has the upper hand here, since traditional logic comprises Aristotelian metaphysics. Therefore the question — are the above propositions interchangeable? — easily gets out of hand.

One final aspect of the relinquishing of the epistemological role in favor of the purely methodological one should be mentioned here. Boolean logic allows a great deal of freedom in the choice of conclusions derivable from a given class of premises. Traditional logic determines for the user one conclusion for each pair of premises by means of fixed scheme of inference (syllogisms). Thus, logic turns from a consultative and instructional system into a neutral system, in which ordinary mathematical intuition is free to operate: logic suddenly turns into a garden of forking paths along which one may wander at will, as long as one takes care not to stray. Traditional logic, by contrast, is a strict tour guide who purports to lead one towards a sublime destination, when in truth this destination is so sublime that it lays beyond the guide’s ken.

5. The historical context of Boole’s achievements does not explain them.

My intent is to conclude this paper with a discussion of the philosophical background to Boole’s extensionalism. Before that, let me briefly provide some comments on its historical and mathematical context. 50 years ago William Kneale fixed the official history of Boole’ s logic by neatly pointing out two major developments in mathematics that served as the background to Boole’ s discovery:

> “(i) [...] there could be an algebra of entities which were not numbers in any ordinary sense, and (ii) [...] the laws which hold for types of numbers up to and including complex numbers need not all be retained together in an algebraic system not applicable to such numbers”.[19]

This statement of fact is indisputable. Rapidly growing, and constantly improving studies have since elaborated it.[20] It should be stressed, however, that historical context of intellectual achievements cannot explain them. The mathematical atmosphere in Boole’s time was important, and perhaps even indispensable to his achievements. In more than one respect his work is one of the many manifestations of the ideological revolution that his friends and peers had affected. But his achievements are not a mere import of techniques from the frontier of mathematics into the forgotten domain of logic (that consequently acquired sudden brilliance and a valuable renaissance). Failing to notice this is failing to notice Boole’s contribution to the philosophy of logic.

As is well known, Leibniz and his followers engaged in a rather nasty quarrel with Newton and his followers over the priority of discovery of the calculus. This quarrel had an important outcome: at least until 1800, English mathematicians used Newton’s notation exclusively, while French and German ones used Leibniz’ s notation. France was at that period the undisputed center of study of calculus, and this had to do with the notational gap in the following way**.** Lagrange’s theory discussed abstract coordinates rather than geometrical ones, and the Newtonian notation was not capable to denote with ease derivation by different coordinates and translations between them.

Consider some important examples. The notion of velocity in Newton's theory was that of (the limit of) the quotient of distance and time. Velocity was not an independent parameter there: Newton's did not think of the term 'velocity' as an abstract coordinate. This holds ever more so for momentum, which is the product of velocity and mass. Lagrange's theory was considerably more liberal: it allowed momentum to be a coordinate just as distance or time. Dismissing all this and much more as mere French finesse, the British intellectual leadership began to lose touch with the frontiers of calculus studies.

In the teens of the 19th century, influenced by Robert Woodhouse's *Principles of Analytical Calculation*, three Cambridge mathematics students revolted against the growing seclusion. They initiated a mini-cultural revolution. It centered on the adaptation and learning of French mathematics. While pretending to be revolting against mere notation, they actually imported updated mathematical research into the British arena and initiated its progress. Their leaders were Charles Babbage (who was possibly the first to design a computer), Sir John Herschel (who was the chief cartographer of the southern sky, an astronomer and a physicist) and George Peacock (who later went into theology, not before writing his influential *Treatise on Algebra* in 1830). In 1812 they established a society for the promotion of their goals. It was called 'The analytic society'. It had an immediate and a tremendous success. It returned to Cambridge its status as an important mathematical center.

Calculus studies of that period were characterized by a tendency towards algebra. One of the central reasons for this was a rather straightforward attempt to assimilate, as much as possible, the Calculus into algebra. In a series of brilliant (regrettably neglected) essays, J. 0. Wisdom showed that this tendency had originated in and was motivated by the desire to answer the criticisms of the Calculus which already Berkeley raised regarding Newton's apparently inconsistent use of the very notion of the differential. In a nutshell, Berkeley claimed that the differential (now designated by the Leibniz delta sign) as used by Newton, conceals a contradiction: within the mathematical computation it designated a negligible magnitude as both unequal and equal to zero. A letter, quoted by Wisdom, from W.R. Hamilton to De Morgan[21], shows just how much the most advanced mathematician of Boole' s time considered Berkeley's criticism in need of an answer (almost a hundred and fifty years after Newton's publication of his *Principia*, and over a hundred years after Berkeley's publication of his *Analyst*).

The exact meaning of Algebraization was not yet understood, for formal Algebra was not fully developed, nevertheless, it was deemed a weapon against possible hidden contradictions: it offered the hope that its use will bring about a calculus as rigorous as possible since algebra was deemed more secure — more basic and more exact. As we know now, the venture was partly doomed to failure and partly successful. Its result, however, was an impressive gain to both the calculus and algebra, chiefly because the venture boosted the tendency towards the abstract and the formal. Gradually the tendency prevailed and algebra came to be presented as a class of formal computation rules on given abstract sets. Signs designating formal computation were then deemed 'empty', since they were abstract. This tendency was in diametric opposition to Newton's insistence on the view of the calculus as a part of geometry. The analytic society imported the notions of formality and abstractness into England and then gradually expanded them. The products of the best of England's mathematical minds of the mid-19th century England (Arthur Cayley, Augustus De Morgan, F. D. Gregory, Sir William Hamilton, James Joseph Sylvester, and many others) can all be regarded as fruits of this development.

This is a partial and brief outline of what is generally (and justly) regarded as an indispensable background to the study of the origins of Boolean algebra (and of twentieth-century mathematics in general). Yet, I contend, even a highly detailed and superbly researched study of this type will lack a critical edge. It will provide us with knowledge of techniques and tools that were at Boole' s disposal as he revolutionized logic. Yet it will not bring us nearer to understanding the philosophical implications of his revolutionary logic. And it will not bring us nearer to understanding its underlying epistemological motives. This is my task here.

It is easy to overlook Boole's contribution to philosophy in a post-Fregean era. But he lived before Frege, of course. In his time logic was commonly regarded as a stagnant relic of a dead tradition. The few attempts that were made to revive it were often hopelessly conservative (Whately, Mill), or a problematic mixture of tradition and modernity (Hamilton and even De Morgan). Most mathematicians neglected logic and even ridiculed it. Why did one of the brightest mathematical minds of the time deem it worthwhile to convert this archaic relic into a shining new algebra? And what changes has this brought to our conception of logic, to our understanding of its scope and function?

The most common way to treat these questions is to submerge them in the Baconian myth that conflates the detailed writing of history with the understanding of its intricacies, This myth blocks the study of motives, for example, as it has no room for problems, and for problem situations. The question regarding Boole's motives, for example, is submerged by the sound of the marching troops of mathematical progress rather than by reference to the problems that he wished to see solved[22]. The myth makes discoveries almost inevitable, given the inevitability of progress.

As Agassi pointed out in his 1963**,** Duhem clarified the myth: he emphasized that pointing at a given tool is by itself a satisfactory explanation of its revolutionary use, simply because history is a constant march towards inevitable progress. Thus, new tools will, sooner or later, be implemented and used as means for solving problems. This explanatory tradition, I contend, has submerged an important aspect in Boole's revolution: its philosophical background, and its philosophical importance. I have tried to explain the latter in the previous section, now let us discuss the former.

6. BOOLE'S SIMPLIFICATION OF THE PROBLEM OF INDUCTION.

In the best British tradition, and as has been pointed out by Musgrave's 1972 paper, Boole believed that thought progresses by the use of concepts. How are concepts generated? Locke contended that they evolve naturally as results of experience. As opposed to him, Leibniz contended that they are inborn. Both answers are unsatisfactory. What they have in common is that they obfuscate the problem they come to solve. The problem they answered is, "How do we generate concepts?" The problem they were trying to solve is, "How do we generate *correct*[23] concepts?" This requires some explanation.

When are we justified in viewing a particular case as demonstrating a general one? How do we choose the generalization (or abstraction, if you will) that is illustrated by the case at hand so as to demonstrate its adequacy? And when is one general concept justly considered as illustrating another (even more general) concept? How do we decide of two different concepts, which is the more general? What is generality? Obviously a case deemed particular can be regarded as illustrating different concepts; the same particular case may even be regarded as illustrating competing generalities (or abstractions). How, then, are we to decide that a given particular concept stand for what general case? The choice here has to be founded on a criterion that separates the correct from the incorrect. But how are we to choose a criterion?

Take, for example, the concept of "swan". Locke contended that this concept is a natural outcome of our encountering swans. This begs our question: On what ground do we determine that an object of perception is a swan? Why do we choose the concept of "swans~~ rather than "northern hemisphere swans," or "white, beaked animals", or "animals with more than one wing", "This" or "Martha, and/or a swan"? Are some concepts more legitimate than others? If so, Why?

And on whose authority? Does the whiteness of several swans justify our view of “whiteness” as an inseparable (essential) part of the concept of swan? Leibniz contended that the concept of swan is simply identical with a list of a-priori attributes (including whiteness). But how do we determine which attributes are necessarily included in the concept and which are merely accidental? (After all, black swans did force us to delete whiteness from swanness.) Is essentialism, with all its inherent difficulties, inevitable here?

This problem is that of the justification of induction. Essentialism is merely an attempt to subdue the problem by conflating epistemology with methodology. Locke and Leibniz were indeed searching for the true sources of true knowledge with the intention of finding explicit justification. And indeed the problem of justifying the induction of informative theoretical knowledge is concerned not with its source but with its justification.

It is easy to associate source with justification. This has been done since time immemorial. Plato said that the source of all knowledge is the Good, and that knowledge is justified by virtue of this reliable source. (Descartes and Leibniz called this reliable source “God”). However, since all of them, Locke and Leibniz included, recognized the very existence of both good and bad generalizations, and since they did not agree on what source is reliable, the problem of the justification of induction lifts its ugly head, To understand the great contribution of Boole here it is essential to notice that the problem has two ugly heads (each head poses a different question). First, what is the distinction between correct and incorrect concepts? Second, when correct concepts are used, what is the distinction between good and bad correlations between concepts?

This is where Boole and his innovative attempt to solve the problem of induction come into the picture. First, Boole contended that all concepts are equally legitimate, since concepts select classes and all classes are legitimate. A class is a legitimate collection of objects of one kind or another, however arbitrary it may be. Therefore the concept of "swan" is just as legitimate as "purple swan" or "swans and/or Martha". Secondly, he contended, we observe Nature and examine the different possible correlations between stipulated objects and possible classes. For example, we may discover that the class of purple swans is empty, that the class of swans is a subclass of white things, and that the class "swans and or Martha" is almost a subclass of the class of white things, since most of its members are white yet one of them is pinkish. Boole didn't stop here. He devised a theory of probability to treat such cases. His aim was to determine to what extent the items of class a, i.e. tentative abstraction a, can be identified with the items of class b, i.e. tentative abstraction b. He thought that in this way he would be able to resolve the question to what extent it is legitimate to view the concepts represented by the classes as equal in meaning. He also demonstrated that the foundations of the theory of probability that he improved comprise a simple but powerful additional interpretation of the innovative logic that he formulated. Finally, he claimed that Boolean logic, ("the pre-inductive stage of science," as he called it) was a description of the universal rules of thought.

Boole, then, held that the mind was open, if only in principle, to accepting any concept as possible and any pairing of concepts as possible in principle. In other words that any conjecture is a possible truth. All the same, science selects the most reasonable conjecture from among all the possible conjectures by means of quantifying observations with the help of the universal rules of thought. The contention that all concepts are legitimate eliminated, if only at first glance, the problem that I have presented here (how can we determine which concepts are correct or legitimate?). In this way the problem of providing a sound basis for knowledge became simpler; it became exclusively the problem of matching concepts by means of observing the particulars that compose them, whatever our conceptualizations of these particulars may be.

Boole’s proposal was to use the theory of probability in order to answer the question, which alternative is the correct scientific theory? This, of course, is largely a traditional answer, for the idea of quantifying observations into probabilistic generalizations and using it to solve the problem of induction was not new and had already been refuted by flume. His refutation, it seems, had not received due recognition. What is new and original in Boole’ s proposal stems directly from his belief that extensionalism is the solution for simplifying the problem: he was, thus, the first logician who could propound the abstract proposition: if a certain particular has the property x (at least approximately) then it also has the property y (at least approximately), without discussing the properties themselves and without entering, not even to the slightest extent, into any metaphysical discussion concerning the nature of properties, the nature particulars, and the nature of the world which inhabits such particulars and properties.

Traditional essentialism (Platonic or Aristotelian) and its traditional opposition, nominalism, had erected insurmountable hurdles here: they had given precedence to ontological discussions over epistemological ones and fused both. In this way they had imposed the acceptance of metaphysical decisions about the make-up of the world even before they had begun investigating it. What is a swan? Is the class of swans and or Martha natural? Is “purple swan” a legitimate term? Does it stand for a legitimate concept? Does it correspond to a genuine entity? According to traditional essentialism, such questions had to be answered before the study of logic could start. Boole has no need for all this. As an extensionalist, his ontology is curtailed to the minimum known to be possible. Should methodology assume from the start that we know which properties are legitimate and which are not? Should it assume that we know which kinds are natural and which are not? Which are the essences and which are the accidents? Boole is the first whose meta-logic does not require an affirmative answer — or any endower -- to these questions.

This is his contribution to the very formulation of the problem of induction.

Nominalism failed to take the discussion forward on this point no less than essentialism. It too put ontology before epistemology. Traditional nominalism sternly forbade any mention of properties or even of classes, simply because it denies the existence of properties and of classes. Therefore, traditional nominalism required some theory of association as the secret glue holding general concepts together. Hence it had to be admitted a priori. Nominalism did not acknowledge general concepts at all, only particular concepts functioning as general ones by the virtue of an a priori association rule. All this was clearly unsatisfactory, since the a priori rule of association was a silent, circular solution to the problem of justifying the "correctness" of "correct" general concepts. Yet, apparently, empiricists had no better option. Even the best of all nominalists, David flume himself, who wouldn't have dreamt of discussing ontology before epistemology, even he did not know of a better option. Nominalism has to concede that the very existence of logic depends on the success of the (highly unsatisfactory, a priori) theory of association, which silenced a formulation of the problem of induction at the conceptual level.

Is Boolean extensionalism a mature and open-eyed version of nominalism? No. Extensionalism is a nominalism pro tem. It lets us talk about classes and properties with the intention of eliminating the precedence of metaphysics over logic, not of eliminating metaphysics as such. (Hume was in error here.) Extensionalism breaks the taboo according to which only graduates of metaphysics are permitted to enter the gates of logic.

Thus, logic was extricated both from metaphysical confusions and from its dependence on the determination of the essences that the world is allegedly comprised of, or the determination of legitimate concepts, or the natural kinds. The new formulation of the problem of induction was this: Given any two classes of objects, are all members of one class also members also of the other?

With what probability? According to the principle of extensionality, this formulation is equivalent to the following: Given two properties, does possession of one of them mandate possession of the other? With what probability? By combining extensionalism with the (psychological) assumption that the rules of logic are the rules of thought, Boole hoped to give logic back its standing as epistemology, for he hoped to have success in explaining how we infer the laws of Nature from observations. Of course he had not solved the problem of induction. There is no problem about the process of induction. The reason for the so called "problem" is a difficulty in accepting the basic fact which expresses one of our great advantages: we are able to produce incompatible conjectures in attempts to explain a given body of data. Today, contrary to Boole's aspiration but directly influenced by his achievements, logic is a real methodology and no epistemology. Therefore his revolution in logic is a classic example of an impressive partial achievement in the overall failure of traditional epistemology.

Dibner Institute, MIT, 2002

BIBLIOGRAPHY

J. Agassi, *Towards an Historiography of Science*, <<History and Theory>>, 2, 1963.

G.P. Baker, P.M.S. Hacker, *Frege: Logical Excavations*, Oxford, Blackwell, 1984.

N. Bar-Am, *A Framework for a Critical History of Logic*, <<Sudhoffs Archiv>>, Band 87, 2003, Heft 1, p. 80-89.

G. Boole, *The Mathematical Analysis of Logic*, Bristol, Thoemmes, [1847] 1998.

G. Boole, *The Laws of Thought*, London, Macmillan, 1854.

G. Boole, *Studies in Logic and Probability*, LaSalle, Open Court, 1952.

G. Boole, *Selected Manuscripts on Logic*, ed. by I. Grattan-Guinness, G. Bornet, Berlin, Birkhauser, 1997.

J. Corcoran, *The founding of Logic*, <<Ancient Philosophy>>, 14, p. 9-24, 1994.

J. Corcoran, *Aristotle's Prior Analytics and Boole's Laws of Thought*, <<History and Philosophy

of Logic>>, 24, p. 261-288, 2003.

A. De Morgan, *On Probability*, London, Baldwin and Cardock, 1830.

A. De Morgan, *Formal Logic*, Ed. by A.E. Taylor, London, Open Court, [1847] 1927.

A. De Morgan, *Syllabus of a Proposed System of Logic*, London, Walton and Maberly,1860.

G. Frege, *Posthumous Writings.* Ed. by H. Hermes, F. Kambartel, Trans. by P. Long, R. White, Oxford, Blackwell, 1979.

J. Gasser, *A Boole Anthology*, Dordrecht, Kluwer Academic Publishers, 2000.

W. Kneale, *Boole and the Revival of Logic*, <<Mind>>, 57, p. 149-175, 1948.

L. Kruger, L. J. Daston, M. Heidleberger, *The Probabilistic Revolution*, Cambridge MA, MIT Press, 1987.

C.I. Lewis, *A Survey of Symbolic Logic*, New York, Dover, [1918] 1960.

C.I. Lewis, C.H.Langford, *Symbolic Logic*, New York, Dover, 1959.

D. MacHale, *George Boole: His Life and Work*, Dublin, Boole Press1985.

A. Musgrave, *George Boole and Psychologism*, <<Scientia>> 107, p. 593-608, 1972.

V. Peckhaus, *Logic, Mathesis universalis und allgemeine Wissenschaft*, Berlin, Akademie Verlag, 1997.

G.C. Smith, *The Boole-De Morgan Correspondence 1842-1864*, Oxford, Clarendon, 1982.

R. Whately, *Elements of Logic*, London, Mawman, 1826.

W. Whewell, *The Philosophy of the Inductive Sciences II*, London, Rutledge, [1840] 1996.

J.O. Wisdom, *The Analyst Controversy: Berkeley's Influence on the Development of Mathematics*, <<Hermathena>>, 54, p. 3-29, 1939.

J.O. Wisdom, *The Analyst Controversy: Berkeley as a Mathematician*, <<Hermathena>>, 59, p.111-128, 1942.

E. Zeller, *Greek Philosophy*. Trans. by S. F. Alleyne, E. Abbot, New York, Henry Holt Company, 1889.

---

[1] I would like to express my gratitude to the *Physis*' readers for their kind comments and helpful criticism.

[2] Boole's explicit introduction of the class Nothing is found in his 1854 p. 52, (quoted in the motto to this paper). In 1847 p. 21, Boole introduces a rudimentary version of the class Nothing as part of his formalization of the open sentence "No Xs are Ys". He says: "Now all individuals common to those classes are represented by XY. Hence the proposition that No Xs are Ys, is represented by the equation XY=0." It is, of course, impossible to determine to what extent Boole realized that he was thus admitting into logic the class Nothing. On the one hand he clearly

acknowledged classes with no members as an integral part of his logic -- see also p. 70 ibid. where 0 clearly depicts a set v' of (some) Ys that are not Xs and that do not exist -- on the other hand he declares, on page 21, somewhat cautiously, that "To assert that no Xs are Ys is to assert that there are no terms common to the classes X and Y". This seems to imply that only terms with reference can exist. It is better to say, then, that like so many ingenious innovators, Boole was only gradually appreciating the far-reaching implications of his innovations.

[3] De Morgan, 1847 [1926] p. 41-42. The classical concept of a sum of all chances is vague, and it is often stated that it may be different than 1 (See for example Bernoulli's *Ars Conjectandi* Part four, Chapter III). A very interesting collection of notes on the history of probability and its state at the first half of the 19th centaury is found in De Morgan 1830

[4] De Morgan 1847 [1926], p. 64.

[5] De Morgan 1860, p. 37-42.

[6] Ibid.

[7] Boole 1847, p. 21. Earlier, yet on the same page, the sign "0" appears for the firs time: Boole formalizes the open sentence "All Xs are Ys" as xy=x and then immediately transform it into x(1-y) = 0 (with no explanation, other than the trivial algebraic knowledge which he seems to assume that his readers share). As Kneale and Laita have noted this "0" seems to be closer to the algebraic zero than to anything like the class Nothing.

[8] Volker Peckhaus maintains that Leibniz has anticipated the index law. (This claim appears in his 1997, in section 5.2.1 and is mentioned in his contribution to Gasser 2000 p. 275-276). This is an example of unhelpful anachronism. Leibniz's outstanding place in the history of logic aside, we should note that his partly successful attempts to express logic by arithmetical notation did not achieve his goal, mainly because he belonged to a tradition that routinely conflated extension and intension as part of its rationalistic, psychologistic epistemology. This, incidentally, explains Leibniz's insistence that synthetic (i.e., informative) truths are impossible: he maintained that all truths are intentional and, as such, analytic (and thus quite unintentionally as Kant as argued, also not really informative). Of course, the index law makes little sense in such a setting: it becomes the logical form of all truths, necessary and contingent. Boole's index law is unique since it is presented in a strictly extensional setting, which is almost naïve. This insured that informative, contingent truths and necessary (analytic) truths are safely kept apart.

[9] *Posterior Analytics* 71a7-28. The point is explicated on 72a. See also next note.

[10] Aristotle is the father of the division of definitions into nominal and real (See his *Posterior Analytics,* Book II, Chapter 10), but his division is highly problematic and caused more confusion than clarification. His notion of nominal definition is strikingly modern. However, his notion of real definition conflates synthetic content and logical necessity in an attempt to secure the epistemic status of the first principles. This point is very subtle and cannot be discussed here in detail. It is intimately connected to Aristotle's theory of induction. Aristotle's justification of the status of first principles (and indeed, his theory of induction) is more aprioristic than has been admitted so far. Especially his insistence that the very act of observing a particular (say, Socrates) already implies knowledge of the universals to which it belongs, and which it manifests (him being a Human, a Mortal, a Greek, etc.,). Aristotle seemed to have believed that the very perception of a particular as such, assumes knowledge of first principles. See, for example, the opening remarks of the *Posterior Analytics* 71a7-28, and their explication on 72a. Possibly this is the first

known transcendental proof for scientific knowledge: it is the claim that without first principles observing particulars would be impossible.

[11] There is no better evidence for the need to clear the situation than J. Corcoran's 2003 comparison of Aristotle and Boole. My claim, that Boole's extensionalism is a crucial step in the process of separating logic from traditional metaphysics, is in direct opposition with his claim that "Aristotle was the founder of logic as formal epistemology and….Boole was the founder of logic as formal ontology" (p. 286). I reject the first part of his statement: I stress the *partially* formal nature of Aristotle's logic. As to what Corcoran says about what he calls "formal ontology", it is hardly worth opposing, since this "formal ontology" is simply a part of formal semantics that shares little or nothing with the traditional metaphysics (Aristotelian or other), which is at issue between us. Boole was indeed one of the fathers of formal semantics (though clearly Leibniz and Bolzano preceded him in many interesting respects). My disagreement with Corcoran is not semantic. It is directed against his double standard for evaluating Aristotle and Boole. He grants his Aristotle, "the founder of formal epistemology", the full protection through a 21st centaury reconstruction of his work, one that has little to do with the spirit or the aims of the original Aristotelian text. And he criticizes Boole by the highest standards possible. His assault on Boole is a topsy turvy, not to say comic, as is his comparison between these two giants that is its basis: "…where Aristotle had a method of deduction that satisfies the highest modern standards of soundness and completeness, Boole has a semi-formal method of derivation that is neither sound nor complete." (p. 261). Indeed, it looks as if Corcoran deliberately ignores formal, sound and complete, reconstruction of Boole's work, such as Schroedr's 19th centaury one, while advocating misleadingly clean 20th centaury reconstructions of Aristotle's work. He refers to some parts of Boole's work as "confused attempt", "misguided effort", "forgotten foray" and finally, "blindness" (ibid), whereas he neatly veils Aristotle's occasional confusions and sometimes-misguided efforts by the rather presumptuous title "formal epistemology". Nor is Corcoran idiosyncratic: Aristotle's work is repeatedly hailed as strikingly modern using contemporary terminology that Aristotle could scarcely have understood. As he certainly was the greatest logician who has ever lived, he does not need defenses that comprise terminological exercises and sleights of hand. Clearly, showing his occasional confusions and his sometimes-misguided efforts is no attack on his greatness. Those who resort to misleadingly clean readings of his work in order to present it as great are the ones who truly belittle it.

I will conclude this digression by noting briefly two examples to the informal and partial nature of Aristotle's logic that Corcoran seems to be unaware of. Many others are available to Aristotle's readers. The first example is well known: it has to do with the fact that Aristotle's notion of a term is imbued with metaphysical presuppositions that are far from being formalized (or even coherently formalizable). Aristotle divides terms roughly into 3 groups: proper names, such as "Socrates", abstract names, such as "Cats", or "Black" and super-abstract names, or categories, such as "substance", "quantity" and "quality" (he declared that there are exactly 10 categories,). Sometimes he further divides the second group of terms into two distinct groups: he distinguishes plural names such as "Cats" or "Blacks" on the one hand, and still more abstract terms such as "Cat" – meaning the idea or notion of a cat – or "Black" – meaning the idea or notion of blackness – on the other hand. The shift between the two is highly problematic and creates a fundamental problem in Aristotle's logic. For example, the inference "Cats are black, black is beautiful, therefore cats are beautiful" seems to be valid, yet the inference "Cats are black, black is a color, therefore Cats are a color" seems to be invalid. Both inferences seem to share a logical form, in Aristotle's system, unless the two senses of Black – the plural or extensional and the intensional – are clearly differentiated and conflating them is prohibited by strict formal rules. Obviously, Aristotle does not commit the fallacy of regarding a valid inference invalid, or vice versa (obviously, Boole does not commit such a fallacy either). However, it is just as obvious that there is no complete set of formal roles in Aristotle to prevent such a fallacy form occurring (and there is none in Boole). This makes the distinction between valid and invalid inferences in Aristotle only partially formal. There are many interesting solutions to this problem in Aristotle, but it is crucial to note that they are all post-Aristotelian. Ascribing them to Aristotle is, at best, unhelpful anachronism. No less odd, but less noted, is the fact that Aristotle explicitly declares that the conclusion in a syllogism could not function as a premise in other syllogisms: having been derived from first principles, it cannot itself be a first principle. And only first principles are allowed as premises! (*Posterior analytics* 71b 26-36). This (together with the regular definition of a syllogism as having only two premises) clearly makes any chain of reasoning (that is longer than two first principles and a conclusion) impossible. Virtually all modern reconstruction of Aristotle's logic ignore this fact (because it is incompatible with what we recognize logic to be). Certainly it limits the formal freedom of Aristotle's logic to that of a set of well-chosen examples (for, clearly, there are no formal rules to the finding of first principles, and to repeat, it makes all chains of reasoning impossible). Corcoran's Aristotle, however has provided us with "…a method of deduction for establishing validity of arguments having unlimited numbers of premises…"

[12] My aim here is to contrast traditional Aristotelian logic with Boolean logic. I will therefore ignore the many interesting differences between "Aristotle's logic", "Aristotelian Logic" the so called "Old Logic", the so called "New Logic" "Scholastic Logic" etc. My aim is to stress that Boole's extensionalism is a breakthrough (and one that is only fully realized in hindsight) in that it is the first logical apparatus that was intended to permit a clear-cut distinction between epistemology and methodology. From this perspective all traditional systems of logic embed what Heinrich Scholz (1961: p. 14-17) called "non-formal logic" and "expanded formal logic" i.e. they embed moves, assumption and frameworks whose aim is to guaranty that, when all is said and done, episteme will be mastered. "Non-formal logic" is, perhaps, not the best choice of a name for it is supposed to complement "formal logic. Kant used the term "formal logic" with respect to his understanding of what Aristotelian logic is…possibly he invented the term. But his use is misleading as Alberto Coffa has noted. First, Aristotle's logic is not "formal" in the modern sense of that word (though, of course, some central parts of it can be formalized, and some central parts of it have already been partially formalized by Aristotle). More importantly, however, Kant himself tends to conflate the psychological aspect of analyticity with the formal one, as Coffa notes. This, I maintain, is a result of the psychologism he inherited from Leibniz and Wolff (see note 8).

[13] Aristotle has his own complex terminology, deliberately avoided here. An example to what I endeavored to put aside is the following. Some scholars maintain that Aristotle's intention in constructing his logic was not to deal with inferences at all but with tautological conditionals of a certain type while others maintain that his original aim was to construct a system of natural deduction. I fail to understand the appeal of such a controversy: it is clearly anachronistic to assume that the modem distinction between inferences and conditionals (or the distinction between an axiomatic system and a system of natural deduction) was, or could have been, observed by Aristotle to any notable measure

(particularly if we observe that his logic is a logic of terms, which is slightly less anachronistic then the above, and if we observe that Aristotle's notion of a term conflates the modern notion of a term and the modern notion of a sentence). Sometimes Aristotle seems to maintain that the syllogism is a theory of sound inference rather than a theory of valid inference. This creates much confusion. It is easy to re-formulate the former possibility to fit the first anachronistic interpretation as follows: Aristotle's theory discusses tautologies whose antecedents are necessary truths. It is interesting to note, however, that it makes little sense to formulate this claim again in terms of the second anachronistic interpretation, that of natural deduction (though it is of course logically possible): the whole point of natural deduction is that it enables us to completely ignore the truth-value of the sentences discussed. Possibly, then, the secret of the popularity of this last interpretation of Aristotle's "true" intention is its ability to silently bypass Aristotle's problematic preference of sound inferences as his subject matter. (See also note 15.)

[14] This fact is closely related to Aristotle's "existential import". In modern logic courses it is presented as an unwarranted, but rather harmless, permission to derive existential sentences from universal ones. This is a reading whose very narrow anachronistic character is of no concern for the authors of the introductory texts, who present it, as their concern is formal logic and not its history. The only existence that it imposes is in its denial that the universe of discourse is empty, not in any prohibition of the use of terms that do not designate an essence. Aristotle's main point, here, is radically different, it is that terms without an essence as reference are to be banned from the realm of logic. Hence all sentences within logic discuss entities which are known to exist. Thus, "existential import" is an anachronistic tag-name for Aristotle's conflation of ontology and logic, and should be understood as such. Modern logic rarely discusses empty domains, and it allows a derivation of existential sentences from universal ones, only through the mediation of a non- empty proper name. This, of course, is not to say that modern logic endorses Aristotle's existential import.

[15] An example of just how confusing the Aristotelian text is on this point is this. Dialectical refutation in Aristotle (as well as in Plato of course) is not merely a display of rhetorical supremacy, but also, and more importantly, a method for searching for the truth. And induction (epagoge) seems to have been initially a tag name for a family of dialectical modes of argumentation (closely linked to the analogy and the example). As such a tag name induction was sometimes not differentiated neatly from the dialectical game itself, in Aristotle. Indeed, Aristotle often applied parallel descriptions to induction and to dialectics, and he even made things less clear by insisting that the Socratic method (which is of course the dialectic method) is induction. Thus, Zeller notes that according to Aristotle: 'The central point of the inquiries which Socrates carries on with his friends is always the fixing of concepts, and the method by which this object is attempted is induction by dialectics" (Zeller 1889 p. 106). What is "induction by dialectics"? Is it a deductive process or an extra-logical one? How is it performed? This is not clear. However, in the opening remarks of his Prior Analytics Aristotle famously notes that the syllogism is the (logical) form that is shared by dialectical inferences and by epideictic (i.e., sound) inferences. (Pr. An. 24a23 -24 b13). This sentence makes perfect sense to us, if we understand it to mean that (dialectical) refutations and sound inferences share a logical form (the valid inference). Valid inferences transmit truth from premises to conclusion in sound inferences, and they transmit falsity from conclusion to the premises in (dialectic) refutations. However, this seems like an anachronistic modernization of the Aristotelian text. It flatly contradicts Aristotle's many definitions of the syllogism as a sound inference rather than a

merely valid one, and it does not explain how induction is to play a role here. If however, we remember that induction and dialectics were not sharply distinguished we may find ourselves noting with Zeller that: "Aristotelian logic (in the "Second Analytics") deals with induction as well as proof but both are preceded by the doctrine of the syllogism, which is the form common to both". (Zeller 1889 p. 182) Zeller, then, ascribes to Aristotle the theory that induction has the logical structure of a valid inference, that it shares its logical structure with sound inferences, that it is a distinct use of the deductive form, and that it somehow overlaps with dialectical refutation.

[16] *Posterior Analytics* 72b 26-33

[17] Even the well-known rubberstamp valid inference "All Greeks are mortals, Socrates is a Greek, Therefore Socrates is mortal." is not really valid, as Lukasiewicz, has noted. For, it includes the term "Socrates," which is a proper name (a name of a primary substance) and hence not *the* name. This is why Leibniz insisted that proper names denote essences: so that we could show that "Caesar has crossed the Rubicon" is a tautology had we known the *definition* of the proper name (of the essence) of Caesar.

[18] I refer, of course, to the famous Scottish logician, a contemporary of Boole, not to be confused with the much more famous Irish mathematician, also a contemporary of Boole. Boole's enigmatic relationship with the latter is discussed in the interesting chapter 13 of MacHale 1985.

[19] Kneale 1948. p. 160

[20] A closed society of a rather questionable eminence dominates the study of the history of logic. It ignores outstanding work that is done outside its ranks, and hails inferior work that is done by its friends and relations. One of its notable products is the anthology on Boole edited by James Gasser 2000. Kneale's classic paper is not included there. No less peculiar is the absence of Alan Musgrave's valuable 1972 paper, which has evidently influenced many authors whom Gasser found worthy of inclusion in his volume.

[21] Wisdom 1939. p. 9-11.

[22] Volker Peckhaus goes as far as to state that Boole's intention was merely to present logic as applied algebra. And he adds: *The mathematical analysis of logic* is not a book on the reform of logic, but a book supporting the break-through of a new algebraic method". (Gasser 2000 p. 276). M.-J. Durand-Richard aptly responds to this: "…it would be anachronistic to speak of his logic as applied mathematics. Perhaps it could be said that logic and mathematics are both applications of symbolical calculus, but only on condition of never losing sight of the fact that when Boole speaks of the first principles of such symbolical calculus, he still views them as based on faculties of mind, which are the only means of explaining them." (Gasser 2000 p. 161). I hope that my present discussion helps to strengthen her claim by elucidating some of Boole's philosophical motives, and thus the philosophical significance of his logic.

[23] Clearly, concepts cannot be true or false since they are not propositions. And clearly the terms "correct concept", "incorrect concept" may be misleading, as they are ambiguous. My aim here is to stress that traditionally the difference between concepts and propositions was somewhat blurred, since, traditionally concepts were treated as something like embryonic propositions. As the concept 'swan' is supposed to include the concept 'whiteness', it is somewhat pregnant with the proposition "all swans are white". This tempts one to consider the concept 'swan' to be correct / incorrect / true / false etc., It is tempting even to maintain that a black swan refutes the concept of swan that

implies whiteness. All this is highly misleading, however, as Boole and Frege and Quine have aptly shown.